Resistance and Capacitance in a DC Circuit

Objective

The purpose of this activity is to empirically observe the behavior of the potential across a capacitor within an RC circuit.

Materials

1. Analog leads to Pasco interface
2. Banana-to-banana wires (x3)
3. Fluke digital multimeter
4. Large aluminium component box

Introduction

Consider the circuit in Figure 1. If the switch is placed in position 1, the circuit is complete and includes the battery. At this point, current flows through the resistor and capacitor. The capacitor acts like a charge reservoir. It is capable of holding an amount of charge $Q_{\text{saturation}} = CV$, where $C$ is the capacitance and $V$ is the voltage across the source. As the current flows through the capacitor, the charge builds up in the capacitor. As the amount of charge in the capacitor approaches $Q_{\text{saturation}}$, the capacitor will have less room for additional charge, so the current will become less and less. If one were to keep the switch in position 1 for a long enough time, the capacitor would become (nearly) fully charged and the current would (nearly) cease to flow. The potential across the capacitor $V_C$ is proportional to the charge that has accumulated in it—i.e., $V_C(t) \propto Q_C(t)$; therefore the potential across a charging capacitor increases with time as $V_C^{\text{ch}}(t) = V(1 - e^{-t/RC})$.

Now, suppose the capacitor is fully charged and the switch is placed in position 2. The capacitor will discharge through the resistor. At first, the capacitor will readily release its charge through the resistor—i.e., the current will be relatively high. As time progresses, the capacitor will have less charge to give away; consequently the current will decrease over time. Again, because the potential across the capacitor is proportional to the charge in it, the potential across a discharging capacitor decreases as $V_C^{\text{dis}}(t) = V_0 e^{-t/RC}$, where $V_0$ is the potential across the capacitor when it begins to discharge.
All exponential decays have a characteristic “half-life”. The half-life is the time that it takes for the dependent variable to be reduced by a factor of 2 (half the original value). For both of the cases above the half-life is \( t_{1/2} = RC \ln(2) \) (see Appendix), where \( R \) is the resistance of the resistor. For example, if the current is 1 amp at \( t = 0 \), it will be 0.5 amps at some time \( t = t_{1/2} \) later, 0.25 amps at \( t = 2t_{1/2} \) later, etc...

The potential across the resistor will always be proportional to the current. So, everything discussed above for current should work for voltage in the circuit above.

**Procedure**

You will be measuring two RC decay half-lives. The first will be long enough to measure using a multimeter and a stopwatch. The second half-life will be one that more typically would appear in electronic circuits and will require a voltage sensor and a signal generator, both of which will be provided by the Pasco 750 Interface (Figure 2).

**Part 1**

1. Open DataStudio on the desktop of your lab computer.
2. Select “Create Experiment”.
3. Click on the yellow circle on the far right of the picture of the interface. A “Signal Generator” window pops up. Click on the highlighted “Sine Wave” drop-down menu, scroll up and select “DC Voltage”.
4. Set the DC Voltage to 2.00 V.
5. Move the Signal Generator window panel over to the right-hand side of the screen (to get it out of the way without closing it).
6. Insert a red banana-banana wire into the far right jack on the interface. This will be the positive (+) terminal of the voltage source.
7. Insert a black banana-banana wire into the ground jack immediately adjacent to the positive terminal. This will be the ground terminal of the voltage source. For now we’re finished setting up the interface.
8. Set up the circuit shown in Figure 3a on page 3. You’re now ready to make measurements of the half-life of the RC circuit.
9. As you click Start (in Data Studio) begin to visually monitor the RUN TIME in the timer panel. Record the time and voltage every 15 seconds for 5 minutes.

10. (Read and understand this entire step before you begin this step!) Remove the wire from the positive terminal of the interface, and immediately plug it into the negative side of the capacitor, shorting the circuit\footnote{In this context, “short” means that you’ll remove the voltage source from the circuit, essentially moving the switch in Figure 1 (on page 1) from position 1 to position 2 by connecting the wire (that was connecting the capacitor to the voltage source) directly to the capacitor. This forms a loop between the resistor and capacitor.}, and immediately start recording the time and voltage every 15 seconds (again) for 5 minutes.

11. Plot the data from steps 9 and 10 on separate plots. Find an experimental half-life from both plots.

12. Measure the resistance of $R$ with a multimeter (disconnect the resistor from the circuit first!). Using this measured resistance and your measured half-life (from step #11) get an experimental value for capacitance of $C$. The manufacturer claims that the capacitance of this capacitor is 1 F (with a 20\% tolerance). Does your result support with this claim?
Part 2

In this part you’ll be using a square wave to act like the battery and a switch from Part 1. To the capacitor, the input voltage will appear to be continually switched from position 1 to position 2.

1. For this part of the experiment, we also need the interface to record the voltage across the capacitor. To include this, in the Data Studio window click on Analog Channel A to “Choose sensor or instrument...”. Scroll to the bottom, and click “Voltage Sensor”, then “OK”.

2. Set up the circuit as illustrated in Figure 3c (on page 3).

3. In the Signal Generator window panel, change the signal generator from “DC Voltage” to “Positive Square Wave”.

4. Set the amplitude to 5.000 V. Set the frequency such that the period of the square wave is significantly longer than your expected RC decay half-life. Why is a longer period important? (Ensure the sampling frequency is large enough.)

5. In the Displays window panel, double-click “Scope”, choose “Voltage, ChA (V)”, and click “OK”.

6. Click Start. If the parameters are all set well, then the trace of the Voltage on the screen will be fairly stable and stationary.

7. Determine the half-life from the oscilloscope trace. Compare this to your calculated value.

Appendix

Derivation of Half-Life Formula

Starting with the definition of capacitance, \( C = Q/V \), where \( Q \) is the amount of charge on each plate (of a parallel plate capacitor) and simply rearranging: \( Q = CV \). Let \( q(t) \) be the instantaneous value of the charge time \( t \) of the capacitor. Then the time dependency of a charging capacitor has the same time dependency as that of the voltage,

\[
q(t) = CV(1 - e^{-t/RC}). \tag{1}
\]

When the charge is half of its maximum value (i.e., for a certain time \( t = t_{1/2} \), \( q(t_{1/2}) = \frac{1}{2}CV \)), the voltage will be half its maximum. If we set \( q(t_{1/2}) = \frac{1}{2}CV \), and solve for \( t_{1/2} \)
in equation (1), then

\[ \frac{1}{2} CV = CV \left( 1 - e^{-t_{1/2}/RC} \right) \]
\[ \frac{1}{2} = (1 - e^{-t_{1/2}/RC}) \]
\[ e^{-t_{1/2}/RC} = \frac{1}{2}. \]

Then taking the natural logarithm of both sides:

\[ -\frac{t_{1/2}}{RC} = \ln \left( \frac{1}{2} \right). \]

Therefore,

\[ t_{1/2} = \ln(2) RC. \]