Graphs

Objectives

The purpose of this experiment is to find relationships between quantities by means of graphing.

Introduction

Graphs are used to show how one quantity depends on another quantity. The relationship between two quantities is not always apparent from looking at the numerical data collected.

Graphs are a means for determining what relationships exist between quantities.

Materials

1. 1-meter stick
2. 30-cm ruler
3. Circular objects
4. String and scissors for room
5. White square blocks

Procedure

In the following exercises we will apply graphing to some familiar situations. In some or all of these cases you will already know the relationship between the variables. But pretend that you do not know this relationship and use graphs to help you discover it. In much experimental work the relationship is not known until the graphs are analyzed.

Donuts and Dozens

Let's start with an idea that is familiar to all of us: donuts. If we go to the bakery and buy one dozen donuts, we expect to have twelve donuts. If we buy two dozen, we have 24 donuts, etc. Take one of the sheets of graph paper and draw two axes. On the horizontal axis (along the bottom side) put numbers from zero to six at conveniently chosen division marks, and label this axis "dozens of donuts". On the vertical axis (along the left side) put numbers from 0 to 72 at conveniently chosen division marks, and label this "number of donuts". **Make sure you choose your scales on the axes so you nearly fill the page.**

We have data (a set of numbers) that look like those in Table 1.

Dozens of Donuts	Number of Donuts
0	0
1	12
2	24
3	36
4	48
5	60
6	72

Table 1: *Donut data*

If we didn't already know, it could be hard to look at a table and see a relationship between the numbers, but a graph of this set of numbers can help us understand the connections among them. Plot the pairs of numbers from the table on your graph. Draw a smooth curve (or straight line if that works) through the points. Although in this case the line will go through all points, in general the idea is not just to connect the points but to draw a smooth curve that goes near points which may have considerable experimental error. You should have a graph that looks like Figure 1. Notice in this case the line through the points is straight. This means the relationship between the two variables on the graph is LINEAR.

It also means we can write a mathematical sentence (equation).

$$y = mx + b$$

where

y is the number of donuts,
m is the slope of the line,
x is the number of donuts (measured in dozens), and
the number of donuts (same units as y) when $x = 0$.

The slope m of this straight line is given by:

$$\text{slope} = \frac{y_{\text{final}} - y_{\text{initial}}}{x_{\text{final}} - x_{\text{initial}}}$$

where x_{initial}, x_{final}, y_{initial}, and y_{final} are *points on the line,* **not necessarily** *data points*[1]. In our donut problem the slope is $[(72-0)\text{donuts}]/[(6-0)\text{doz}]$ or 12 donuts per dozen; no surprise! Since the number of donuts is zero when the number of dozens is zero, i.e. our graph goes through (0,0), our intercept is zero and our mathematical sentence (equation) is:

$$\begin{aligned} \text{number of donuts } &= 12 \text{ donuts/dozen } \times \text{ number of dozens } + \text{ zero} \\ y &= 12x + 0 \end{aligned}$$

[1] *Hint*: Choose points on the line as far apart as possible to get a more accurate value for the slope.

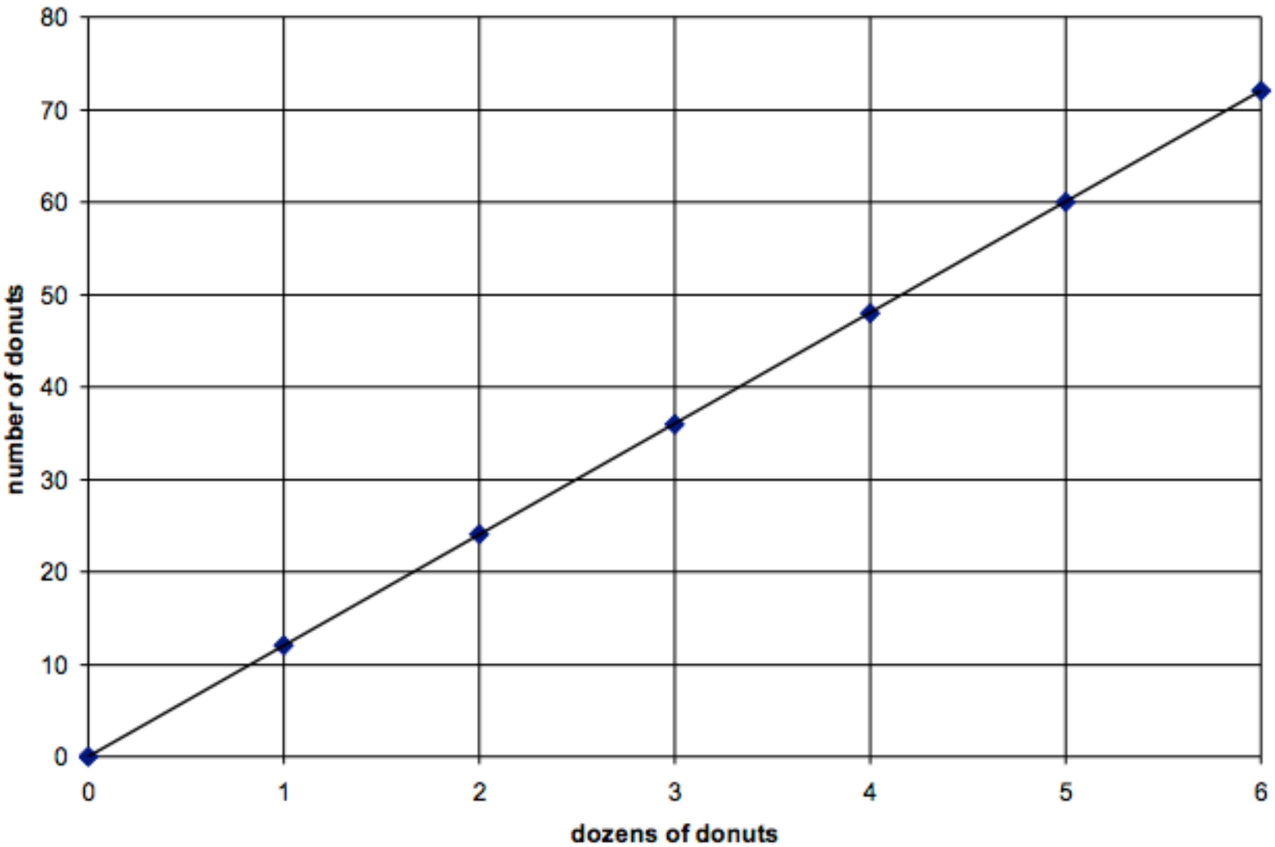

Figure 1: *The Donut Problem*

Area and Length

Now take the squares and measure the length of one side and count the squares marked on each block. Each marked square represents an area of one square centimeter. Make a table showing the relationship between the length of a side and the area. After adding the appropriate columns to your table, plot three curves:

1. area versus length,
2. area versus length squared, and
3. area versus length cubed.

1. (a) Which of these is a straight line?

   (b) What is its slope?

2. (a) How can we write a mathematical sentence that reveals what our graphs show?

   (b) Is this mathematical sentence (equation) one you have seen before?

Important caveats: Remember to draw a single straight line or smooth curve through your data points. No matter how strongly you are tempted, do not just "connect the dots". Also, take great care when picking points from which to calculate slope. While it is not technically incorrect, you should **not** use points from your data set; it is permissible *only if* they fall *exactly* on your line (but it is a bad habit!)[2]. In Figure 1 you *could* use the points (0, 0) and (5, 60) even if they *are* data points; this is so *only* because they are on the best-fit line.

Circular Objects

Measure the radius and the circumference of each of the circular objects. One strategy for getting the circumference is to place a string around the object and then to measure the length of string required. For the same circular objects now measure the areas of the circles. You can do this by tracing the outline on graph paper with 1 cm by 1 cm squares. Count the enclosed squares, estimating fractional squares. (You should be able to conserve paper by putting all circles on one sheet.)

Plot circumference vs radius, circumference vs radius squared, and circumference versus square root of radius.

3. (a) Which is a straight line?
   (b) What is its slope?
4. (a) From your prior experience with circles can you tell what this slope is?
   (b) What mathematical sentence (equation) gives the relation between circumference and radius?

Plot area versus radius, area versus radius squared, and area versus square root of radius.

5. (a) Which is straight?
   (b) What is its slope?
6. (a) What is the mathematical sentence (equation) for this linear graph?
   (b) Do you remember the well-known name of the slope in this case?

Note: Be sure that you make a 5-column data table for the variables in your graphs (one column for radius, circumference, area, r^2, and $\sqrt{r}$.

[2]Your data set allows you to construct a best-fit line. Once you have a best-fit line, you should ignore your data set. In this particular activity, you will have quite precise data, so the best-fit line will almost certainly pass through all of your data; in general, however, this will not be the case.