Graphs

Objective

The purpose of this activity is to learn and develop some of the necessary techniques to graphically analyze data and extract relevant relationships between independent and dependent phenomena, and to communicate those relationships to others.

Materials

1. Access to spreadsheet program (Microsoft Excel is installed on laboratory desktops)
2. Graph paper (optional)
3. Ruler (optional)

Introduction (Graphing Techniques)

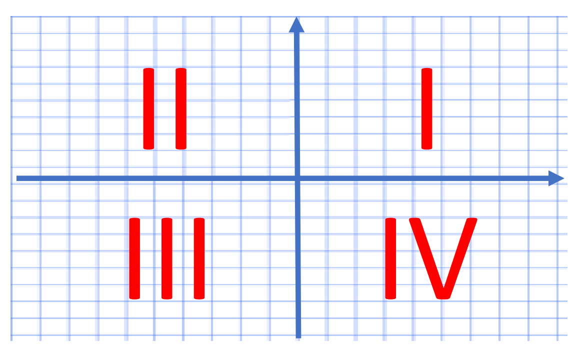
A graph often gives a clear indication of how one physical quantity depends on another. In drawing graphs (especially if graphing by hand on graph paper) consider the following ideas:

1. ***Scale*** -- Choose scales that use most of the sheet. However, pick a scale that is also convenient. If three squares represent 10, each square is 3 1/3 . This is not convenient. A more convenient choice would be to let one square represent 2 or 5 depending on which best utilizes most of the sheet. Unless you actually have data near zero for a variable, you do not always have to start your axis at zero. For example, the left end of your horizontal axis might be 0 or 5 or -3 depending on the range of values of interest to you.

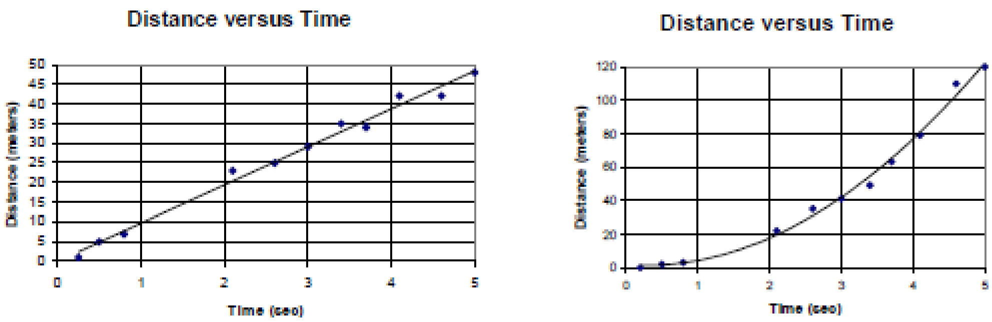
2. ***Labels*** -- Put a descriptive title at the top of the graph. Label the axes with the variable names or symbols. Put numbers at enough divisions to make the scale clear; continue numbering with the same intervals between numbers all along the axis. Clearly indicate the units for each axis.

3. ***Data points*** -- Plot points neatly. You may wish to make points easier to see, and distinguish them from other sets of points on the same graph, by surrounding each by a point protector, a small triangle, square, circle, etc.

4. ***Quadrant*** -- We can select the quadrant based on the data. For example:
   - If all values are positive, we use quadrant I on the graph paper.
   - If y-values are positive values and x-values are negative then we choose quadrant II.
   - If all values are negative, we choose quadrant III.
   - For negative y and positive x, we choose quadrant IV.



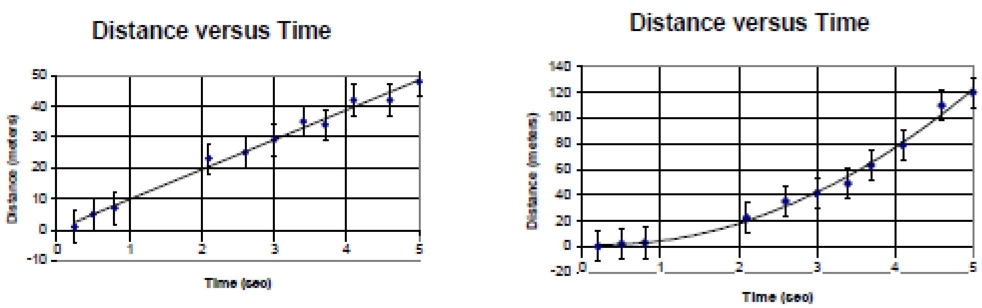
5. ***Trendline*** -- Draw a smooth line or curve through the data points that you have plotted. Do not just connect the points|do not \connect the dots"! Nature usually produces smooth variations, but experimental data usually have some random scatter.
   Since you do not know which points are most accurate, draw your smooth line with about as many points above it as below it. Figures 1a and 1b each contain graphs of possible distance vs time data. The data in Figure 1a are clearly represented most easily by a line, while it is just as clear that the data in Figure 1b are not.



(a) (b)

Figure 1

6. ***Uncertainties*** -- If you know or can estimate the uncertainties in your data, use error bars to indicate a range of values rather than a precise point, as demonstrated in Figures 2a and 2b, which represent the same data from Figures 1a and 1b now with error bars.



(a) (b)

Figure 2

7. ***Slope***[1] -- For instances in which the slope of a graph is the quantity of interest, you should draw a straight line tangent to the graph at the point of interest. Then choose two points on the

[1] Physicists often generically refer to the vertical axis as the “y" axis and the horizontal as the “x" axis, but these axes are usually given specfic names, as with the graphs in the figures here. In some cases, one of the axes will be called “x" but will not be the horizontal axis, as in some of the examples the problems below. In physics we use symbols to mean more than one thing; for example “x" can refer to either the generic horizontal axis or to the vertical axis of a graph of position versus time (a common graph in physics). Do not naturally assume that “x" values go on the horizontal axis.

tangent line you just drew. For convenience, we'll call these two points (x_1, y_1) and (x_2, y_2). Choose these two points such that they are (a) as far apart as reasonably possible (to reduce error) and (b) as near to gridlines as possible, making their values easier to discern (for convenience). Then draw horizontal and vertical lines to complete a right triangle. The slope m is the ratio of the vertical side to the horizontal side|i.e., $m = (y_2 - y_1)/(x_2 - x_1) = \Delta y/\Delta x$. The slope is negative if the tangent line is tilted down to the right.

In this course you will usually calculate the slope of a straight line, rather than the instantaneous slope of a curve. While the principle is the same, you must take care when choosing points for slope calculation when dealing with a straight line. If, for example, you were calculating the slope of the straight line in Figure 1a and you used the fourth and fifth data points for your slope calculation, you would be calculating the slope of a line different from the one graphed above (try drawing a line through those two points for illustration). On the other hand, if you use the second and fifth data points, you would get the right value for the slope of that line. For this reason, after you have drawn the best fit line, ignore the data points on the graph. Choose (x_1, y_1) and (x_2, y_2) as two points on the line; do not choose data points.

8. ***Finding relationships*** -- Although there are some very complex relationships in nature, very often one variable will depend simply on a power of another. For example, if $y = mx^k + b$, a graph of y vs x will be a curve but y vs x^k will be a straight line with slope m and y-intercept b. When trying to determine a relationship from experimental data, if y vs x does not give a straight line (to within experimental uncertainty), try graphs of y vs x^2, y vs $x^{1/2}$ ($= \sqrt{x}$), etc. Don't forget about negative powers. If y decreases as x increases, you may need to consider y vs x^{-1}(= 1/x) or y vs $x^{-1/2}$($= 1 / \sqrt{x}$), etc. With experience and some thought you'll know which has the best chance of giving a straight line. There's always a chance that no simple form will give a straight line.

Practice with Graphs

1. Let's start easy. The data in Table 1 were chosen to correspond closely to a straight line. Represent uncertainties in the y values by error bars. For example, at x = 3, y is between 8 and 10. Draw the best straight line on your graph; compute its slope and intercept. Write an equation for y as a function of x.

x	y
-1	-6.5 ± 1.0
2	$+5.0 \pm 1.0$
3	$+9.0 \pm 1.0$
4	$+16 \pm 2$
7	$+26 \pm 2$
8	$+29 \pm 2$

Table 1: *Table for use with Part 1*

2. The data in Table 2 gives position in meters versus time in seconds for a moving particle. The uncertainties are negligible. Plot x vs t. Compute the slope at t = 1 sec. What are the units for

this slope? Is the slope changing?

Next you're going to use a pre-made spreadsheet to plot the data in different ways. Specifically, in this case, you're going to plot x vs t^k, where k is some value of your choosing|for example, . . . , -2, -1/2, 1/2, 2, . . . , etc. Open the spreadsheet entitled, **GraphsfindingExponents.xls** on the computer's desktop. Navigate to the tab labeled "No 2". You'll notice that the graph changes as you change the value of k in cell G2. Change the value of k until the graph looks like a straight line. (For this lab, just try integers and the reciprocal of integers.) Once you determine an appropriate value for k, record the values of t^k, then draw the graph by hand. (Don't forget to properly label the axes, give a title to your graph, etc.!) As before, draw the best straight line on your graph; compute its slope and intercept. Write an equation for x as a function of t.

t (sec)	x (meters)
0.3	0.2
1.5	4.5
2.1	8.8
3.5	24.5
4.2	35.3
5.4	58.3

Table 2: *Table for use with Part 2*

3. Table 3 contains realistic data for the average speed of air molecules as a function of absolute temperature -- i.e. the number of degrees above absolute zero. Room temperature is about 300 kelvins. From your graph determine the approximate speed of an air molecule in this room. Try to determine the functional dependence of molecular speed on temperature (using \No 3" tab on the spreadsheet). Again, after determining the best value for k, record the values of Tk, and plot your linear graph by hand (properly labeled, of course!). Assume all speeds to be about 5% uncertain.

T (kelvins)	v (m/s)
40	180
90	290
150	360
210	420
290	510
340	530

Table 3: *Table for use with Part 3*

4. Finally, Table 4 contains data that indicates what happens when ultraviolet or visible light "knocks" electrons from the surface of a material. KE_{max} is the maximum energy with which the electrons "fly" off, and λ is the wavelength of the incident radiation. Do not be concerned that the units of energy, electron volts (eV), may be unfamiliar. The units of wavelength are nanometers (nm); 1 nm = 1 x 10^{-9} meters. By finding a way of plotting the data that will yield a straight line (with the help of the "No 4" tab on the spreadsheet), determine the relationship between the maximum energy and the wavelength. What is the slope? What are the units of the slope? What is the intercept, and what are its units? Write an equation (with appropriate units on the constants) for maximum energy as a function of wavelength.

About 100 years ago a careful analysis similar to what you are asked to do here contributed to the discovery of the quantum nature of radiation.

λ (nm)	KE_{max} (eV)
200 ± 20	3.9 ± 0.2
280 ± 10	2.1 ± 0.1
340 ± 10	1.4 ± 0.1
390 ± 10	0.9 ± 0.1
440 ± 5	0.50 ± 0.05
480 ± 5	0.30 ± 0.05
520 ± 2	0.10 ± 0.02

Table 4: *Table for use with Part 4*

Before you leave, close the spreadsheet file, and do not save the changes.