Phase and Impedance in an RLC Circuit

Objective

The purpose of this experiment is to introduce you to some of the phenomena and terminology of AC circuits involving resistance, inductance, and capacitance. In making measurements you will use the digital storage oscilloscope and signal generator capabilities of the Pasco 850 computer interface.

Figure 1

Materials

1. Berkeley coil (solenoid)
2. Fluke Digital Multimeter
3. Large aluminnum component box
4. Long wires with banana leads
5. Pasco 850 Interface

8. Regular banana leads
9. Solenoid (concentric with iron rod core)
10. Voltage probes (x3)

Procedure

By means of banana-plug-tipped wires connect the series RLC circuit shown in Figure 1a. For the source, you will use the signal generator on the far-right of the Pasco 850 interface (marked "1"). Use $R = 1\,k\Omega$ and $C = 1\,\mu F$. For the inductance, L, use the large copper coil on the clear plastic spool. The three analog channels of the interface should have voltage probes, each consisting of red and black wires tipped by banana plugs. Connect these probes so that they will measure the voltages across R (connected to channel A), L (connected to channel B), and C (connected to channel C). To make your voltages have the correct sign, you need to maintain polarity by connecting each black lead to the "side" of the circuit element "nearest" to ground.

The Oscilloscope/Computer Setup

When your connections are complete and the computer is on, use the following computer operations to activate and adjust the oscilloscope:

1. Open Pasco Capstone on the desktop of your lab computer.
2. In the "Tools" windows panel on the left side of the screen, click on "Hardware Setup". Click on the yellow circle around analog "Channel A" and select "Voltage Sensor". Repeat for channels B and C.
3. On the top far right of the interface marked "1", click on the yellow circle and select "Output Voltage-Current Sensor".
4. Click on "Signal Generator" in the "Tools" window panel. Select "850 Output 1" and confirm that the waveform is set to "Sine". Set the voltage ("Amplitude") to 1 V and frequency to 100Hz. Click "On" to turn on the signal generator.
5. In the "Displays" window to your right, double click on "Scope". Click on "Select Measurement" on your vertical axis and choose "Voltage, ChA(V)". To add plots for channels B and C, click on the y-axis ("Voltage "Ch A"); hover your mouse over "Add Similar Measurement" and select or check the second and third "Voltage" respectively. This should give you a scope that will display all three voltages on the y-axis and time on the x-axis.
6. In the "Controls" window panel at the bottom of your screen, change the sampling mode to "Fast Monitor" then click "Monitor" to begin.
7. Adjust your axes to have a clearer view of your scope. (Zoom into/out of your y-axis by clicking your mouse and hovering it up or down and having the maximum amplitude/voltage in mind.)
8. Adjust the sampling frequency to obtain smooth sinusoidal waveforms for the three channels.
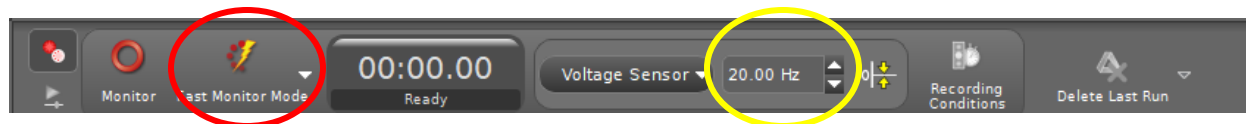
Figure 2: *"Controls" Panel. The yellow circle indicates the sampling frequency while the red circle shows the recording mode. (Click the arrow to change the mode)*

At this point you should carefully determine and write down the channel and color of trace that corresponds to each of the circuit elements. Later you will need to know at a glance which voltage you are observing. Click the "Trigger" ⬆ button to stabilize your scope.

You should now have a relatively stable display with each new trace being triggered by a rise in the A signal. Vary the amplitude and frequency of the signal generator and note the change in the display. Notice what device dominates at high frequencies and what dominates at low frequencies. Why do you think this happens (explain in your report)? Now set the signal generator back to 1 V and 100 Hz, and hit stop.

Phase Measurements

Each of the voltages in your circuit (v_R, v_L, v_C, and v_S, the source voltage) has the form $v(t) = V_{max}$ $\sin(\omega t + \phi)$, where V_{max} is the amplitude (the maximum value, the zero-to-peak voltage) (see Figure 1b). The phase is $\omega t + \phi$, where ω is the angular frequency in radians per second. $\Omega = 2\pi f$, where f is the frequency in hertz. Note that the time needed for one cycle or period ($T = 1/f = 2\pi/\omega$) corresponds to $2\pi = 360°$. On the oscilloscope this is the time from one peak of a trace to the next peak on that same trace. Because ϕ is different from one circuit element to the next, their voltages are not in phase—i.e. not in step; they do not reach peak values at the same time. For example, a difference in phase of $\pi = 180°$ would correspond to a time difference of $T/2$; one signal would be half a cycle ahead of the other, one would have a peak when the other has a valley.

At the top of your scope click the "Delta" ⊹ tool. This will help you track data points and display their coordinates. Now answer the following questions:

1. What is the period, T ? What is the frequency, $f = 1/T$? Is this value consistent with your setting on the signal generator?

2. What is the time delay between a peak of V_R and the closest peak of V_C? What about V_L and V_R? V_L and V_C?

3. What phase difference, in radians and degrees, does each of these time delays imply?

4. Given that the current is the same for all circuit elements and that for the resistor $V = IR$, we have:
$$I(t) = v_R(t)/R.$$

   This means that the current is in phase with the voltage across the resistor. What is the phase difference between current and the two other voltages that you are observing? Express your answer in the form "V leads I by $X°$" if the peaks of V precede the peaks of I, or "V lags I by $X°$" if the peaks of V follow the peaks of I.

Impedance

The definition of resistance is $R = V_R/I$. This applies to either instantaneous values or maximum values of current and voltage. Therefore,

$$R = V_{RM}/I_M, \tag{1}$$

where I_M is the peak current, and V_{RM} is the peak voltage across the resistor. *Reactance* and *Impedance* are defined by analogy with this equation. Inductive reactance and capacitive reactance are defined by

$$X_L = V_{LM}/I_M, \text{ and } X_C = V_{CM}/I_M, \tag{2}$$

respectively. Reactance can be related to capacitance by

$$X_C = 1 \ / 2\pi \ fC, \tag{3}$$

Similarly, inductive reactance is

$$X_L = 2\pi \ fL, \tag{4}$$

where L is the inductance measured in enries (H).

Measure the peak values of your three voltages. Just in case the "zeroing" of the traces was not complete, measure peak-to-peak – i.e., the difference between the positive and negative peak voltages – and take half. With these values use equation (1) to obtain I_M . Then use (2) to obtain the reactances. As a check, use equation (3) to obtain C. Does this value agree with the capacitance you connected into the circuit? Use equation (4) to obtain L. The total impedance connected to the source (analogous to the total resistance in a DC circuit) is

$$Z = V_{SM} \ / I_M \tag{5}$$

For a series RLC circuit, when the phase differences are properly accounted for, it can be shown that

$$Z = [R^2 + (X_L - X_C)^2]^{1/2} \tag{6}$$

Use R and your values of X_L and X_C in equation (6) to compute Z, and use equation (5) to compute the peak source voltage, V_{SM} . Click on the y axis ("Voltage") and select "Output Voltage" to monitor it in your scope. Click "Monitor" then "Stop" after a few seconds. Determine the peak value of this source voltage. How does this peak value compare with the value you just calculated?

Phase Angle for RLC

A final quantity that is often specified for an RLC circuit is referred to as the phase angle (ϕ). It is not to be confused with the "phase" or "phase shift" of a specific circuit element.
Phase angle refers to the whole RLC circuit; it is defined by the equation

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

A positive phase angle corresponds to a circuit that is more inductive in nature and in which the current lags behind the voltage, while a negative phase angle corresponds to a circuit which is more capacitive in nature and in which the current leads the voltage. A popular way to remember this is the saying "ELI the ICE man," which shows the phase relationship between voltage (E) and current (I) for either an inductor (L) or a capacitor (C). *Determine the phase angle for your circuit using the values of R, X_L and X_C.* Now measure this phase angle directly by comparing the relative positions of the output voltage (v_s) trace to the trace for the voltage across the resistor (v_R). *Compare the two values for the phase angle*

and explain any difference. Specifically, does the sign of the measured phase angle agree with the expected sign—i.e., positive or negative?

Now remove the output voltage trace from your scope by unchecking "Output Voltage" from the "Add Similar Measurement" option. Reactivate the oscilloscope to receive new signals (hit "Monitor"). Observe what happens to your voltages when you place a large piece of iron inside the inductor. What happens to each of three signals? What does this tell you about the inductance of the coil when the iron core is added? Estimate the new value for the inductance. Remove the iron before going on.

Resonance

According to equation (6), the impedance should be a minimum when $X_L = X_C$. For a fixed V_{SM} this should mean maximum current (resonance). Making use of the fact that the two reactances are frequency dependent, you could vary the frequency to find the value that gives maximum current (as evidenced by the maximum case for a voltage across a resistor, $V_R = IR$). This would be the frequency which makes $V_{LM} = V_{CM}$ (or $v_L(t) = v_C(t)$). However, specific resonance frequency can be difficult to find by trial and error. So, using this $X_L = X_C$ idea for resonance, set equation (3) equal to (4) and solve for this frequency. Now go to your signal generator window and type in this frequency. Hit "Monitor" and see if, indeed, your capacitor and inductor have the same amplitude and if $v_L(t) = v_C(t)$ (this will appear as two waves that are 180° out of phase); hit "Stop". Were you correct? If you switch the black and red leads on your inductor (channel B) then $v_L(t)$ should appear the same as $v_C(t)$. Activate your scope and check this. Do they match (do they now appear to be in phase and have the same amplitude)?

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